Nonlinear distortion in simple transistor stages:

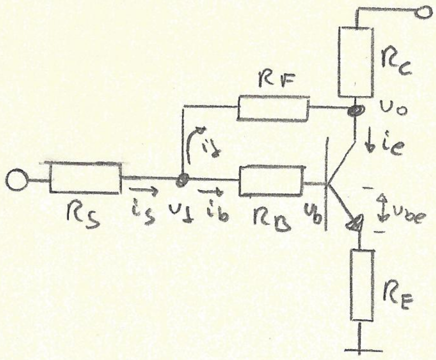

Fig. 1

1. Expansion of the base-emitter voltage current relation into a Taylor series.

(1) $I_E = I_0\left(e^{\frac{U_{BE}}{V_T}} - 1\right)$, where $V_T = \frac{kT}{q}$, k is Boltzman's constant, T is the absolute temperature in Kelvin and q is the charge of an electron.

Capital letters indicate quiescent voltages and current, small letters indicate signal components.

(2) $I_E + i_e = I_0\left(e^{\frac{U_{BE}+u_{be}}{V_T}} - 1\right)$

(3) $i_e = (I_E + I_0)\left(e^{\frac{u_{be}}{V_T}} - 1\right)$

which gives for u_{be}:

(4) $u_{be} = V_T \ln\left(\frac{i_e + I_E + I_0}{I_E + I_0}\right)$

and $I_0 \sim 0$

(5) $u_{be} = V_T \ln\left(\frac{i_e + I_E}{I_E}\right) = V_T \ln\left(1 + \frac{i_e}{I_E}\right)$

which gives the serie (Appendix A)

(6) $u_{be} = V_T\left(\frac{i_e}{I_E} - \frac{i_e^2}{2I_E^2} + \frac{i_e^3}{3I_E^3} \dots\right)$

as long as $-1 < \frac{i_e}{I_E} < 1$

2. The circuit equations for fig. 1

(1) $U_{in} = u_{RE} + u_{be} + i_b \cdot R_b + i_s \cdot R_s$ (2) $u_f = v_b + i_b \cdot R_b$

(3) $u_{RE} = i_e \cdot R_E$ (4) $i_s = i_b + i_f$ (5) $v_o = -(i_e - i_f) R_C$

$$i_f = \frac{u_f - v_o}{R_F} = \frac{u_f + i_e R_C - i_f R_C}{R_F}$$

$\Downarrow$

(6) $$i_f = \frac{u_f + i_e R_C}{R_F\left(1 + \frac{R_C}{R_F}\right)} = \frac{u_f + i_e R_C}{R_C + R_F}$$

(7) $$v_{in} = v_{be} + i_e\left(R_E + \frac{R_s + R_B}{\beta}\right) + \frac{\left(U_{be} + i_e\left(R_E + R_C + \frac{R_B}{\beta}\right)\right) R_s}{R_C + R_F}$$

$\Downarrow$

$$V_{in} = U_{be}\left(1 + \frac{R_s}{R_C + R_F}\right) + i_e\left[R_E + \frac{R_s + R_B}{\beta} + \frac{(R_E + R_C) R_s}{R_C + R_F} + \frac{R_b \cdot R_s}{\beta (R_C + R_F)}\right]$$

$\Downarrow$

(9) $$v_{in} = u_{be}\left(1 + \frac{R_s}{R_C + R_F}\right) + i_e\left[R_E + \frac{(R_E + R_C) R_s}{R_C + R_F} + \frac{R_s + R_b + \frac{R_b \cdot R_s}{R_C + R_F}}{\beta}\right]$$

$\Downarrow$

(10) $v_{in} = U_{be} \cdot A_1 + i_e R_1$

From (1.6):

(11) $$U_{in} = A_1\left[i_e \cdot r_e - \frac{r_e i_e^2}{2 I_E} + \frac{r_e i_e^3}{3 I_E^2}\right] + i_e R_1$$

$\Downarrow$

(12) $$V_{in} = i_e(r_e \cdot A_1 + R_1) - \frac{r_e A_1}{2 I_E} i_e^2 + \frac{r_e A_1}{3 I_E^2} i_e^3$$

From (5) and (6)

$$U_o = -\left[i_e \cdot R_C - R_C \cdot i_e \frac{R_E + R_C + \frac{R_B}{\beta}}{R_C + R_F} - U_{be} \cdot \frac{R_C}{R_C + R_F}\right]$$

$\Downarrow$

$$U_o = -\left[i_e R_C\left[1 - \frac{R_E + R_C + \frac{R_B}{\beta}}{R_C + R_F}\right] - \frac{R_C}{R_C + R_F}\left[r_e i_e - \frac{r_e i_e^2}{2 I_E} + \frac{r_e i_e^3}{3 I_E^2}\right]\right]$$

$\Downarrow$

(13) $$v_o = -\left[i_e R_C\left[1 - \frac{R_E + R_C + \frac{R_B}{\beta} + r_e}{R_C + R_F}\right] + i_e^2 \cdot \frac{r_e R_C}{2 I_E (R_C + R_F)} - i_e^3 \frac{r_e R_C}{3 I_E^2 (R_C + R_F)}\right]$$

$\Downarrow$

(14) $U_o = -\left[i_e a + i_e^2 b + i_e^3 c\right]$

and $i_e = V_{in} A + V_{in}^2 B + V_{in}^3 C$

$$v_0 = -\left[v_{in} A a + v_{in}^2 B a + v_{in}^3 C a + v_{in}^2 A^2 b + 2 v_{in}^3 A B b + v_{in}^3 A^3 c\right]$$

⇕

(15) $$v_0 = -\left[v_{in} A a + v_{in}^2 \left[B a + A^2 b\right] + v_{in}^3 \left[C a + 2 A B b + A^3 c\right]\right]$$

From (12), we invert this series expansion (App. B)

$$i_e = V_{in} \cdot \frac{1}{r_e A_1 + R1} + V_{in}^2 \frac{r_e A_1}{2 I_E (r_e A + R1)^3} + \frac{\frac{r_e A_1}{3 I_E^2}(r_e A_1 + R_1) - \frac{r_e^2 A_1^2}{2 I_E^2}}{(r_e A_1 + R1)^5} V_{in}^3$$

⇕

(16) $$i_e = V_{in} \cdot A + V_{in}^2 B + V_{in}^3 \frac{r_e A_1 (2 R1 - r_e A1)}{I_E^2 \cdot (r_e A_1 + R1)^5 \cdot 6}$$

⇕

$i_e = V_{in} A + V_{in}^2 B + V_{in}^3 \cdot C$ as in (14)

From (15):

(17) $A_u = A \cdot a$

(18) 2nd: $= V_{in} \cdot \left[\frac{B}{A} + \frac{A b}{a}\right]$

(19) 3rd: $= V_{in}^2 \left[\frac{C}{A} + \frac{2 B b}{a} + \frac{A^2 c}{a}\right]$

From earlier we've made the following definitions:

$$a = R_c \left[1 - \frac{R_c + r_e + R_E + \frac{R_B}{\beta}}{R_c + R_F}\right]$$

$$b = \frac{r_e R_c}{2 I_E (R_c + R_F)} \qquad c = -\frac{r_e R_c}{3 I_E^2 (R_c + R_F)}$$

$$A = \frac{1}{r_e A_1 + R1} \qquad B = \frac{A_1 \cdot r_e}{2 I_E (r_e A_1 + R1)^3} \qquad C = \frac{r_e A_1 (2 R1 - r_e A_1)}{6 I_E^2 (r_e A_1 + R1)^5}$$

$$A_1 = 1 + \frac{R_S}{R_c + R_F} \qquad R1 = R_E + \frac{(R_c + R_E) R_S}{R_c + R_F} + \frac{R_S + R_B + \frac{R_S R_B}{R_c + R_F}}{\beta}$$

This will then give us the following expressions:

$$(20)\qquad A_u = \frac{-R_C\left[\dfrac{R_F - R_E - r_e - \frac{R_B}{\beta}}{R_C + R_F}\right]}{r_e A_1 + R_1}$$

$$(21)\qquad 2nd = V_{in}\left[\frac{A_1 \cdot r_e}{2 I_E (r_e A_1 + R_1)^2} + \frac{r_e}{2 I_E \left[R_F - R_E - r_e - \frac{R_B}{\beta}\right]\left[r_e A_1 + R_1\right]}\right]$$

$$3rd = V_{in}^2\left[\frac{r_e A_1 (2R_1 - r_e A_1)}{6 I_E^2 (r_e A_1 + R_1)^4} + \frac{3 A_1 r_e^2 - 2 r_e (r_e A_1 + R_1)}{6 I_E^2 \left[r_e A_1 + R_1\right]^3 \left[R_F - R_E - r_e - \frac{R_B}{\beta}\right]}\right]$$

$$(22)\qquad 3rd = V_{in}^2\left[\frac{r_e A_1 (2R_1 - r_e A_1)}{6 I_E^2 (r_e A_1 + R_1)^4} + \frac{r_e (r_e A_1 - 2R_1)}{6 I_E^2 \left[r_e A_1 + R_1\right]^3 \left[R_F - R_E - r_e - \frac{R_B}{\beta}\right]}\right]$$

3. Simplified formulas:

Set $A1 = 1$ $\quad R1 = RE + \dfrac{(R_C + R_E)\, R_S}{R_C + R_F}$

$$A_u = -\frac{R_C \cdot R_F}{(R_C + R_F)(re + R1)}$$

$$2nd = v_{in} \frac{re}{2I_E (re + R1)^2}$$

$$3rd = v_{in}^2 \frac{re(2R1 - re)}{6I_E^2 (re + R1)^4}$$

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Appendix A

Serie expansion of ln(1+x)

$$\ln(1+x) = a_1 x + \tfrac{1}{2} a_2 x^2 + \tfrac{1}{3} a_3 x^3 + \tfrac{1}{4} a_4 x^4 + \ldots\ldots \tfrac{1}{n!} \cdot a_n x^n \ldots\ldots$$

The endividual coeffisients are determined by: the formula

$$a_n = \frac{f^{(n)}(0)}{\cancel{}} \quad , n \geq 1 \quad , \text{where } f(x) = \ln(1+x)$$

$$a_1 = f'(0) = \frac{1}{1+x} = 1 \quad (a_0 = f(0) = \ln 1 = 0)$$

$$a_2 = f''(0) = \frac{-1}{(1+x)^2} = -1$$

and

$$a_n = f^{(n)}(0) = \frac{(-1)^{n-1}}{1^n}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \ldots\ldots + \frac{(-1)^{n-1} x^n}{n!} + \ldots$$

If x is substituted with $-x$ ~~will~~ we'll get the series expansion of $\ln(1-x)$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \ldots\ldots + \frac{-x^n}{n!} + \ldots\ldots$$

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Appendix B

Inversion of series

This analysis inverts the 3 first parts of the series.

Given the function:

(1) $$ay + by^2 + cy^3 = x$$

which should be inverted to give:

(2) $$y = Ax + Bx^2 + Cx^3$$

From (1):

$x = 0$ will give $y = 0$

The first derivative:

$$ay' + 2by \cdot y' + 3cy^2 y' = 1$$

Set $x = 0$:

$$y' = \frac{1}{a} = A' \qquad A = \frac{A'}{1!}$$

The second derivative:

$$ay'' + 2b(yy'' + y'^2) + 3c(y^2y'' + 2yy'^2) = 0$$

Set $x = 0$

$$y'' = -\frac{2b}{a^3} = B' \qquad B = \frac{B'}{2!}$$

The third derivative:

$$ay''' + 2b(y'y'' + yy''') + 2b \cdot 2y'y'' + 3c(y^2y''' + 2yy'y'' + 2(y'^3 + 2yy'y'')) = 0$$

Set $x = 0$

$$y''' = \frac{6(ca - 2b^2)}{a^5} = C' \qquad C = \frac{C'}{3!}$$

(3) The series is then $y = \frac{x}{a} - \frac{bx^2}{a^3} - \frac{ca - 2b^2}{a^5}x^3$