Nonlinear distortion in simple transistor stages. Part II
The differential pair.

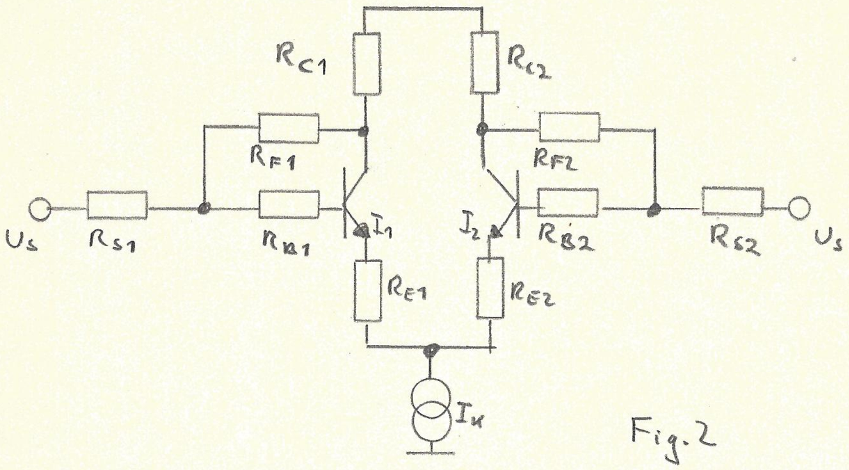

Fig. 2

1. Expansion of the base - base voltage current relation into a Taylor serie

a) Mismatch of the differential pair

The mismatch m is defined as:

(1) $$m = \frac{2[I_1 - I_2]}{I_K} \quad , \text{ where } I_K = I_1 + I_2$$

which gives

(2,3) $$I_1 = I_K \frac{1+\frac{m}{2}}{2} \quad \text{and} \quad I_2 = I_K \frac{1-\frac{m}{2}}{2}$$

(4,5) and is $$I_1 = I_K M_1 \quad \text{and} \quad I_2 = I_K M_2$$

b) The circuit equations for the emitter circuit

(6) $$U_i = U_{be1} + U_{RE1} - U_{be1} - U_{RE1}$$

$$I_{E1} + i_{e1} = I_0 \left(e^{\frac{U_{BE1}+u_{be1}}{U_T}} - 1\right)$$

(7) $$i_{e1} \hat{=} (I_{E1} + I_0)\left(e^{\frac{u_{be1}}{U_T}} - \right)$$

and the same for I_{E2} gives

$$(8) \qquad ie_2 = (I_{E2} + I_0)\left(e^{\frac{U_{be2}}{V_T}} - 1\right)$$

This relations may be inverted to give:

$$(9) \qquad U_{be_1} = V_T \ln \frac{ie_1 + I_{E1} + I_0}{I_{E1} + I_0}$$

and $$(10) \qquad U_{be_2} = V_T \ln \frac{ie_2 + I_{E2} + I_0}{I_{E2} + I_0}$$

and when (4),(5) and $I_0 \sim 0$ is taken into account:

$$(11) \qquad U_{be_1} = V_T \ln \frac{ie_1 + I_K \cdot M_1}{I_K M_1}$$

and $$(12) \qquad u_{be_2} = V_T \ln \frac{ie_2 + I_K M_2}{I_K M_2}$$

We define:

$$(13) \qquad re_1 = \frac{\cancel{4} V_T}{I_K M_1} \quad \text{and} \quad (14) \qquad re_2 = \frac{\cancel{4} V_T}{I_K M_2}$$

The equations (11) and (12) is now on the form wanted in app. A part I, and be expanded into ~~the serie~~, a Tayln serie, and using eq. (6),(13) and (14) will be:

$$(15) \qquad U_i = \frac{re\, ie_1}{\cancel{4}} + ie_1 \cdot R_{E1} - \frac{re\, ie_1^2}{2\cancel{4} I_K M_1} + \frac{re\, ie_1^3}{3\cancel{6} I_K^2 M_1^2} - ie_2 R_{E2} - \frac{re_2\, ie_2}{\cancel{4}} + \frac{re_2\, ie_2^2}{2\cancel{4} I_K M_2}$$
$$- \frac{re\, ie_1^3}{3\cancel{6} I_K^2 M_2^2}$$

(16) We have $ie_1 = -ie_2$

This gives the two useful equations

(17) $$v_i = ie_1\left[R_{E1}+R_{E2}+re_1+re_2\right] + ie_1^2\,\frac{re_2M_1 - re_1M_2}{2I_kM_1M_2} + ie_1^3\,\frac{re_1M_2^2 + re_2M_1^2}{3I_k^2M_1^2M_2^2}$$

and

(18) $$v_i = -ie_2\left[R_{E1}+R_{E2}+re_1+re_2\right] + ie_1^2\,\frac{re_2M_1 - re_1M_2}{2I_kM_1M_2} - ie_1^3\,\frac{re_1M_2^2 + re_2M_1^2}{3I_k^2M_1^2M_2^2}$$

2. The circuit equations for the differential pair shown in fig. 2 and in fig. 3

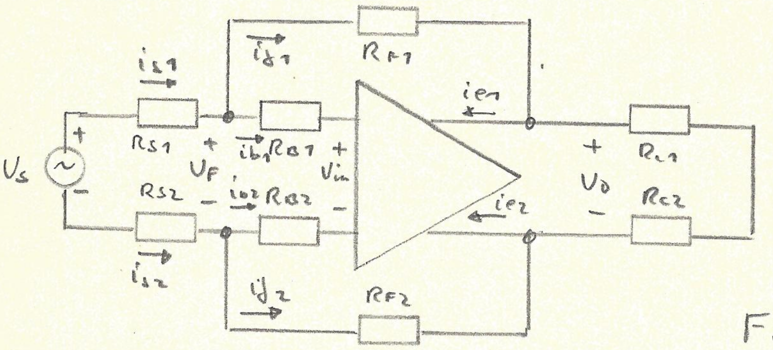

Fig. 3

(1) $U_s = U_{in} + ib_1\cdot R_{B1} + is_1\cdot R_{s1} - ib_2\cdot R_{B2} - is_2\cdot R_{s2}$

(2) $ib_1 = ie_1\cdot\frac{1}{\beta}$, $ib_2 = ie_2\cdot\frac{1}{\beta}$

(3) $\frac{U_s - U_i}{R_{s1}+R_{s2}} = ib_1 = -ib_2$

(4) $U_o = -(ie_1\cdot R_{c1}) + ie_2\cdot R_{c2}$

(5) $U_{o1} = -(ie_1 - if_1)R_{c1}$

(6) $U_{o2} = -(ie_2 - if_2)R_{c2}$ $\qquad U_o = U_{o1} - U_{o2}$

(7) $U_{F1} = \frac{U_{in}}{2} + ib_1\cdot R_{B1}$ $\qquad U_{F2} = ib_2\cdot R_{B2} - \frac{U_{in}}{2}$

(8) $if_1 = \frac{U_{F1} - U_{o1}}{R_{F1}}$ $\qquad if_2 = \frac{U_{F2} - U_{o2}}{R_{F2}}$

(9) $is_1 = if_1 + ib_1$, $is_2 = if_2 + ib_2$

From eq 5-9 will get:

$$(10) \quad if_1 = \frac{ie_1 \cdot R_{C1} + U_{F1}}{R_{F1} + R_{L1}} = ie_1 \left[\frac{R_{C1} + \frac{R_{B1}}{\beta}}{R_{F1} + R_{C1}} \right] + \frac{U_{in} \cdot R}{2(R_{C1} + R_{F1})}$$

The same can be done for ie_2 , and the result when put into eq. (7) is:

$$(11) \quad U_S = U_{in} + \frac{ie_1}{\beta}\left[R_{B1} + R_{S1}\right] - \frac{ie_2}{\beta}\left[R_{B2} + R_{S2}\right] + ie_1 \left[\frac{R_{C1} \cdot R_{S1}}{R_{C1} + R_{F1}} + \frac{R_{B1} \cdot R_{S1}}{\beta(R_{F1} + R_{C1})} \right]$$

$$+ U_{in} \left[\frac{R_{S1}}{2(R_{C1} + R_{F1})} + \frac{R_{S2}}{2(R_{C2} + R_{F2})} \right] - ie_2 \left[\frac{R_{B2} + R_{S2}}{\beta} + \frac{R_{B2} \cdot R_{S2}}{\beta(R_{F2} + R_{C2})} \right]$$

Note:

In eq. (10) we can see that the factor R_{C1} in the denominator $R_{F1} + R_{C1}$ is an expression for the fact that the output is driving the feedback network as a voltage generator with a given output impedance, R_{C1}.

(11) will then be:

$$(12) \quad U_S = U_{in} \left[1 + \frac{R_{S1}}{2(R_{F1} + R_{C1})} + \frac{R_{S2}}{2(R_{F2} + R_{S2})} \right] + ie_1 \left[\frac{R_{B1} + R_{S1}}{\beta} + \frac{R_{C1} \cdot R_{S1}}{R_{C1} + R_{F1}} + \frac{R_{B1} \cdot R_{S1}}{\beta(R_{C1} + R_{F1})} \right]$$

$$- ie_2 \left[\frac{R_{B2} + R_{S2}}{\beta} + \frac{R_{C2} \cdot R_{S2}}{R_{C2} + R_{F2}} + \frac{R_{B2} \cdot R_{S2}}{\beta(R_{C2} + R_{F2})} \right]$$

which can be defined as:

$$(13) \quad U_S = U_{in} \cdot A1 + ie_1 \cdot R1 - ie_2 R2$$

which can be divided into the two equations

(14) $U_s = U_{in} \cdot A1 + ie_1 [R1 + R2]$

and

(15) $U_s = U_{in} \cdot A1 - ie_2 [R1 + R2]$

Equation (14) and (15) can now be combined with equations (1.17) and (1.18) respectively, which will give us:

(16) $$U_s = ie_1 \left[R1 + R2 + A1 [R_{Et}]\right] + ie_1^2 \frac{re_2 M_1 - re_1 M_2}{2 I_K M_1 M_2} + ie_1^3 \frac{re_1 M_2^2 + re_2 M_1^2}{3 I_K^2 M_1^2 M_2^2}$$

(17) $$U_s = -ie_2 \left[R1 + R2 + A1 [R_{Et}]\right] + ie_2^2 \frac{re_2 M_1 - re_1 M_2}{2 I_K M_1 M_2} - ie_1^3 \frac{re_1 M_2^2 + re_2 M_1^2}{3 I_K^2 M_1^2 M_2^2}$$

(17.b) where $R_{Et} = 2 R_{E1} + R_{E2} + re_1 + re_2$

Eq. (16) can be written as:

(18) $U_s = ie_1 \cdot a_1' + ie_1^2 b_1' + ie_1^3 c_1'$

which can be inverted (App. B. I) to:

(19) $ie_1 = U_s \cdot A_1 + U_s^2 B_1 + U_s^3 C_1$

Eq. (17) can be written as:

(20) $U_s = ie_2 a_2' + ie_2^2 b_2' + ie_2^3 c_2'$

which can be inverted to:

(21) $ie_2 = U_s \cdot A_2 + U_s^2 B_2 + U_s^3 C_2$

3. The output circuit

(1) $U_{o1} = -(ie_1 - ij_1) R_{C1}$

$$U_{o1} = -ie_1 \cdot R_{C1} + ie_1 \cdot R_{C1}\left[\frac{R_{C1}}{R_{F1}+R_{C1}} + \frac{\frac{R_{B1}}{\beta}}{R_{F1}+R_{C1}}\right] + \frac{V_{in}}{2} \cdot \frac{R_{C1}}{R_{F1}+R_{C1}}$$

(2) $$U_{o1} = V_{in}\frac{R_{C1}}{2(R_{F1}+R_{C1})} + ie_1 \underbrace{\left[\frac{R_{C1} + \frac{R_{B1}}{\beta}}{R_{F1}+R_{C1}} - 1\right] R_{C1}}_{R_K}$$

and $\frac{R_{C1}}{2(R_{F1}+R_{C1})} = K1$

which can be serieexpanded and written as:

(3) $$U_{o1} = ie_1\left[R_K + K1\, R_{Et}\right] + ie_1^2\, K1 \frac{re_2 M_1 - re_1 M_2}{2 I_K M_1 M_2} + ie_1^3\, K1 \frac{re_1 M_2^2 + re_2 M_1^2}{3 I_K^2 M_1^2 M_2^2}$$

which can be defined as:

(4) $U_{o1} = -\left[ie_1 \cdot a_1 + ie_1^2 b_1 + ie_1^3 c_1\right]$

(5) $U_{o2} = -(ie_2 - ij_2) R_{C2}$

(6) $$U_{o2} = V_{in} \underbrace{\frac{-R_{C2}}{2(R_{C2}+R_{F2})}}_{K2} + ie_2 \underbrace{R_{C2}\left[\frac{R_{C2} + \frac{R_{B2}}{\beta}}{R_{F2}+R_{C2}} - 1\right]}_{RH}$$

which can be serieexpanded and written as:

(7) $$U_{o1} = -ie_2\left[-RH + K2\, R_{Et}\right] + ie_2^2\, K2 \frac{re_2 M_1 - re_1 M_2}{2 I_K M_1 M_2} - ie_1^3\, K_2 \frac{re_1 M_2^2 + re_2 M_1^2}{3 I_K^2 M_1^2 M_2^2}$$

which can be defined as:

(8) $U_{o2} = -\left[ie_2 a_2 + ie_2^2 b_2 + ie_2^3 c_2\right]$

4. Expansion of the nonlinearity describing functions

When U_o is on the form $U_o = -[ie\, a + ie^2 b + ie^3 c]$ and ie on the form $ie = V_{in} A + V_{in}^2 B + V_{in}^3 C$ the system equations for U_o as a function of Q_s. V_{in}, where V_{in} is the source voltage is:

(1) $$U_o = -\left[V_{in} A a + V_{in}^2 \left[B a + A^2 b\right] + V_{in}^3 \left[C a + 2 A B b + A^3 c\right]\right]$$

The gain is then:

(2) $$A_u = -A a$$

The second degree term is:

(3) $$2nd = -V_{in}\left[\frac{B}{A} + \frac{A^2 b}{a}\right]$$

The third degree term is:

(4) $$3rd = -V_{in}^2\left[\frac{C}{A} + \frac{2Bb + A^2 c}{a}\right]$$

When the output is sensed differential, will have:

(5) $$U_o = U_{o1} - U_{o2}$$

When noting a for U_{o1} as a_1 an so on:

(6) $$U_o = V_{in}\left[A_2 a_2 - A_1 a_1\right] + V_{in}^2\left[B_2 a_2 + A_2^2 b_2 - B_1 a_1 - A_1^2 b_1\right] + V_{in}^3\left[C_2 a_2 + 2 A_2 B_2 b_2 + A_2^3 c_2 - C_1 a_1 - 2 A_1 B_1 b_1 - A_1^3 c_1\right]$$

The gain is now

(7) $A u = A_2 a_2 - a_1 A_1$

and the distortion is the second degree term on the first degree term and the third degree terms on the first degree term for the second and third degree nonlinearity factors respectively.

5. Summary of definitions

Main definitions

(1) $$a_1 = R_{C1}\left[1 - \frac{R_{C1} + \frac{R_{B1}}{\beta}}{R_{C1} + R_{F1}}\right] - \frac{R_{C1} \cdot R_{et}}{2(R_{C1} + R_{F1})}$$

(2) $$b_1 = \frac{-R_{C1}}{2(R_{C1} + R_{F1})} \frac{[re_2 M_1 - re_1 M_2]}{2 I_K M_1 M_2}$$

(3) $$c_1 = \frac{-R_{C1}}{2(R_{C1} + R_{F1})} \frac{[re_1 M_2^2 + re_2 M_1^2]}{3 I_K^2 M_1^2 M_2^2}$$

(4) $$a_2 = R_{C2}\left[1 - \frac{R_{C2} + \frac{R_{B2}}{\beta}}{R_{C2} + R_{F2}}\right] - \frac{R_{C2} R_{et}}{2(R_{C2} + R_{F2})}$$

(5) $$b_2 = \frac{R_{C2}}{2(R_{C2} + R_{F2})} \frac{[re_2 M_1 - re_1 M_2]}{2 I_K M_1 M_2}$$

(6) $$c_2 = \frac{-R_{C2}}{2(R_{C2} + R_{F2})} \frac{[re_1 M_2^2 + re_2 M_1^2]}{3 I_K^2 M_1^2 M_2^2}$$

For cases where $R_{x1} = R_{x2}$ ~~and~~ note that

$a_2 = a_1$, $b_2 = -b_1$, $c_2 = c_1$

$$(7) \quad A_1 = \frac{1}{R1 + R2 + A1 \cdot R_{et}}$$

$$(8) \quad B1 = \frac{-A1\left[re_2 M_1 - re_1 M_2\right]}{\left[R1+R2+A1 R_{et}\right]^3 \, 2 I_k M_1 M_2}$$

$$(9) \quad C1 = \frac{A1 \, \dfrac{re_1 M_2^2 + re_2 M_1^2}{3 I_k^2 M_1^2 M_2^2}\left[R1+R2+A1 R_{et}\right] - 2\left[A1 \, \dfrac{re_2 M_1 - re_1 M_2}{2 I_k M_1 M_2}\right]^2}{\left[R1+R2+A1 R_{et}\right]^5}$$

$$(10) \quad A2 = \frac{-1}{R1 + R2 + A1 \cdot R_{et}}$$

$$(11) \quad B2 = \frac{A1\left[re_2 M_1 - re_1 M_2\right]}{\left[R1+R2+A1 R_{et}\right]^3 \, 2 I_k M_1 M_2}$$

$$(12) \quad C2 = \frac{A1 \, \dfrac{re_1 M_2^2 + re_2 M_1^2}{3 I_k^2 M_1^2 M_2^2}\left[R1+R2+A1 R_{et}\right] - 2\left[\dfrac{re_2 M_1 - re_1 M_2}{2 I_k M_1 M_2} A1\right]^2}{-\left[R1+R2+A1 R_{et}\right]^5}$$

For cases where $R_{x1} = R_{x2}$ note that

$A1 = -A2$, $B1 = -B2$, $C2 = -C1$

Sub-definitions

$$(13)\quad R1 = \frac{R_{B1} + R_{S1}}{\beta} + \frac{R_{C1} \cdot R_{S1}}{R_{C1} + R_{F1}} + \frac{R_{B1} \cdot R_{S1}}{\beta(R_{C1} + R_{F1})}$$

$$(14)\quad R2 = \frac{R_{B2} + R_{S2}}{\beta} + \frac{R_{C2} \cdot R_{S2}}{R_{C1} + R_{F1}} + \frac{R_{B2} \cdot R_{S2}}{\beta(R_{C1} + R_{F1})}$$

$$(15)\quad A1 = 1 + \frac{R_{S1}}{2(R_{C1} + R_{F1})} + \frac{R_{S2}}{2(R_{C2} + R_{F2})}$$

$$(16)\quad R_{et} = R_{E1} + R_{E2} + re_1 + re_2$$

$$(17)\quad M1 = \frac{1 + \frac{m}{2}}{2}$$

$$(18)\quad M2 = \frac{1 - \frac{m}{2}}{2}$$

$$(19)\quad m = \frac{2\,[I_1 - I_2]}{I_K}$$

$$(20)\quad re_1 = \frac{V_T}{I_K M_1}$$

$$(21)\quad re_2 = \frac{V_T}{I_K M_2}$$